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Compactified NCOS and duality

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Abstract: We study four-dimensional U(1) on a non-commutative T^2 with rational Θ. This theory has dual descriptions as ordinary SYM or as NCOS. We identify a set of massive non-interacting KK states in the SYM theory and track them through the various dualities. They appear as stretched strings in the non-commutative U(1) providing another example of the IR/UV mixing in non-commutative field theories. In the NCOS these states appear as D-strings with winding and momentum. They form an unconventional type of 1/4 BPS state with the 3-brane. To obtain a consistent picture of S-duality for compactified theories it is essential to keep track of both the NS and the RR B-fields.

Keywords: D-branes, String Duality.

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Contents

1. Introduction

Non-commutative field theories arise as world volume theories on D-branes in constant B-field backgrounds [1, 2]. One may hope that a better understanding of these field theories will lead to a better understanding of these backgrounds in string theory.

Apart from that non-commutative field theories have many unusual properties, that make them interesting in their own right. One of the more intriguing properties of non-commutative Yang-Mills theories is S-duality [3]–[6]. The S-dual of four-dimensional NCYM turns out to be a non-commutative open string theory (NCOS). Such theories are very unusual, since generally poles in open string amplitude require the inclusion of closed strings to ensure unitarity. The only other known example of consistent string theories without gravity are the Little String Theories [7], but unlike the NCOS, these theories have no weak coupling limit which makes them hard to study (see however [8]). The NCOS on the other hand can have arbitrarily small coupling and we can study it in string perturbation theory.

In [9, 10] is was pointed out that in the $(1+1)$-dimensional case the NCOS is dual to an ordinary maximally supersymmetric gauge theory with one unit of electric flux turned on (see also [11, 12]). This provides one example where the exotic NCOS has an equivalent description in terms of an ordinary commutative field theory. In this paper we provide another example of this type. More precisely, we study four-dimensional U(1) compactified on a non-commutative T^2 [13]. If we choose the dimensionless non-commutativity parameter $\Theta = 1/s$, where s is an integer, this theory has an equivalent description as a commutative U(s) theory with one unit of magnetic flux. On the other hand, the original non-commutative U(1) has an S-dual description as a four-dimensional NCOS on a torus. In the $(1+1)$-dimensional case there were only two descriptions of the theory, NCOS and ordinary field theory. In our case there is a third as a non-commutative field theory.

Some features of these theories can be understood from the underlying string description [1, 2]. The non-commutative U(1) theory can be realized as the decoupling limit of a single 3-brane on T^2 with s units of NS B-flux. This B-flux through the torus induces s D-string charges in the non-compact directions. Note that these D-strings are a source for the RR B-field, so in general both the NS and the RR B-field are turned on. T-duality on the torus turns the induced D-strings into D3-branes and the original 3-brane into a D-string. Thus we find a U(s) gauge theory with one unit of magnetic flux. We review the field theory picture of this duality in the next section.

In the commutative Yang-Mills description the gauge group factorizes as $\mathrm{U}(s) = (\mathrm{U}(1) \times \mathrm{SU}(s))/Z_s$, and the U(1) is free. Since the theory is compactified on T^2, we expect a KK tower of massive non-interacting 1/4 BPS states from the reduction of the U(1). It is interesting to identify these states in the other two descriptions of this theory as well.

The KK states of the commutative theory turn into states carrying both momentum and winding in the dual description as a non-commutative U(1). According to [14] an open string state moving in one of the non-commutative directions has a finite extent in the other. This statement is strictly true in the $\alpha' \to 0$ limit, which eliminates the oscillator contributions. However, this is the field theory limit we want to take in the end, so thinking of these open strings as objects with a well-defined length makes sense.

It turns out that the states dual to the KK states of the U(1) carry precisely the required amount of momentum in one compact direction to wrap around the other an integer number of times. This provides another example of the IR/UV connection in non-commutative field theory [15]. Since these open strings wrap a compact direction an integer number of times, we expect that they can turn into closed strings. In section 3 we show that there are indeed closed strings with degenerate mass. However it turns out that all open string states that have integer winding around the compact directions decouple from the degrees of freedom of the non-commutative U(1). We show that this is the case by computing the tree level scattering amplitude for four open string states. If one or more of the string states can turn into a closed string, the amplitude vanishes in the field theory limit.

In section 4 we discuss the S-dual picture. Since the BPS states of interest could be identified with stretched open strings before S-duality, they turn into D-strings in the NCOS. We show that D-strings with the appropriate winding and momentum quantum numbers form a marginally bound 1/4 BPS state with the D3-brane, while all other D-strings form real bound states. These 1/4 BPS states are specific to this theory and do not exist in general. They appear to be a new type of 1/4 BPS state, since there does not seem to be a simple way to dualize them into the familiar D0-D4 system without any fluxes turned on.

In order for the masses of these states to match the masses of the BPS states before S-duality, it is crucial that the NS flux through the torus before S-duality turns into RR flux after the duality transformation. In the original flat space version of this S-duality the RR fields were set to zero [3], but after compactification this is no longer a valid approach, since the winding states are sensitive to these fluxes.

2. Non-commutative U(1) and its Morita duals

We consider four-dimensional U(1) theories on a non-commutative T^2. These theories can be labeled by their gauge coupling, the size of the T^2, and by the non-commutativity parameter, $\Theta = \theta/(2\pi R^2)$. Here T^2 is defined by the identifications $x_{2,3} \to x_{2,3} + 2\pi R$, $[x_2, x_3] = i\theta$, and the other two directions are uncompactified.

The tree level action for this theory is given by [2]

$$S = \frac{1}{g_{\mathrm{YM}}^2} \int d^4x (\hat{F}_{ij} + \Phi_{ij})(\hat{F}^{ij} + \Phi^{ij}) , \tag{2.1}$$

and the one-loop action was recently calculated in [16]. We will be interested in theories with $\Phi = 0$ and $\Theta = 1/s$. The first choice is for convenience, but theories with rational Θ are qualitatively different than theories with irrational Θ. This becomes clearer if one studies suitable Morita equivalent descriptions of this theory. Morita equivalence can be studied entirely within the field theory or it can be viewed as the remnant of the T-duality group of the underlying string realization of these theories [13].

For our present purposes it is sufficient to know that this theory has a $\mathrm{SL}(2,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$ duality group, which is the T-duality group of the T^2. We will be interested in the $\mathrm{SL}(2,\mathbb{Z})$ that acts on the non-commutativity parameter as

$$\Theta \longrightarrow \tilde{\Theta} = \frac{c + d\Theta}{a + b\Theta} , \tag{2.2}$$

where a, b, c, d define an $\mathrm{SL}(2,\mathbb{Z})$ matrix. If Θ is rational this allows us to trade our original theory for an equivalent theory with $\tilde{\Theta} = 0$, i.e. a commutative theory. The other parameters of the theory also transform under this $\mathrm{SL}(2,\mathbb{Z})$:

$$\begin{aligned} &\tilde{\Phi} = (a + b\Theta)\Phi - b(a + b\Theta) , \qquad \tilde{R} = (a + b\Theta)R , \\ &\tilde{m} = am + bN , \qquad \tilde{N} = cm + dN . \end{aligned} \tag{2.3}$$

Here m is the number of magnetic fluxes and N the rank of the gauge group. In our case the parameters of the original theory are $\Phi = m = 0$, $N = 1$, $\Theta = 1/s$, and the $\mathrm{SL}(2,\mathbb{Z})$ matrix that transforms to a commutative theory is given by $a = 0$, $b = -c = 1$, $d = s$. The dual theory has $\tilde{\Phi} = -1/s$, $\tilde{m} = 1$, $\tilde{N} = s$, $\tilde{\Theta} = 0$. These parameters define a commutative $\mathrm{U}(s)$ gauge theory with one unit of magnetic flux, compactified on a torus with radii $\tilde{R} = R/s$.

This theory has been studied in great detail [13]. Here we just note that the $\mathrm{U}(s)$ gauge group factorizes into $(\mathrm{U}(1) \times \mathrm{SU}(s))/Z_s$ and that the U(1) is free at all energies. This in turn implies that there should be a tower of non-interacting KK

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states from reducing the U(1) on the torus. We expect these states to be BPS states with masses given by

$$M^2 = \frac{s^2}{R^2}(n_2^2 + n_3^2)\,, \tag{2.4}$$

where $n_{2,3}$ are the momentum quantum numbers in the two compact directions.

The mass formula for the BPS states of NCYM compactified on $T^2 \times S^1$ is well known [13],

$$M = \frac{g_{\rm YM}^2}{2V\mathcal{N}}(w^i + \Theta^{ij}n_j)^2 + \frac{V}{4g_{\rm YM}^2\mathcal{N}}(m_{ij} + \mathcal{N}\Phi ij)^2 + \frac{1}{\mathcal{N}}\sqrt{\frac{k_i^2}{\Sigma_i^2}}\,, \tag{2.5}$$

where $k_i = \mathcal{N}n_i - m_{ij}(w^j + \Theta^{jk}n_k)$, $\mathcal{N} = N + \frac{1}{2}m_{ij}\Theta^{ij}$, V is the volume of $T^2 \times S^1$ and Σ_i are the radii of the corresponding circles. In these expressions w^i are electric flux quanta and n_i are momentum quantum numbers in the compact directions. The magnetic flux mentioned above is given by $m_{23} = m$.

We can readily identify the KK tower of U(1) states in this expression by setting the parameters in eq. (2.5) to the values corresponding to the commutative U(s) theory. With $\tilde{\Theta} = 0$, $\tilde{w}_i = 0$, $\tilde{N} = s$, $\tilde{\Phi} = -1/s$, $m = m_{23} = 1$, $n_1 = 0$ and $\Sigma_{2,3} = \tilde{R} = R/s$, the BPS mass formula reduces to eq. (2.4).

It is interesting to trace these states to the non-commutative description as well. Using the transformations equation (2.3), we can transform the KK states of the commutative theory into the corresponding states of the non-commutative gauge theory. In the non-commutative case we have $\Theta = 1/s$, $\Phi = 0$, $m = 0$, but $w^i = \tilde{n}^i$ and $n_i = -s\epsilon_{ij}\tilde{n}^j$, so these states carry multiples on s units of momentum in the compact directions. Inserting these quantum numbers into the BPS mass formula we verify that these states have the same mass as the states in the commutative description, which is of course a reflection of the fact that these theories are Morita equivalent. While it was straightforward to see that these states do not interact in the commutative description, it is slightly less obvious in the non-commutative case. In the next section we show that these states do in fact decouple by studying the embedding of this theory in string theory. This may seem like overkill since one can show decoupling of these states within the field theory by expanding the gauge fields into momentum eigenfunctions on the torus. The commutator term in the action vanishes for gauge fields carrying s units of momentum, which implies that these fields are free. However, the string description will be useful when we discuss the S-dual description of this theory.

3. The string description of non-commutative U(1)

The theories discussed in the previous section can be obtained from string theory by taking the decoupling limit [2] of the world volume theory on a 3-brane in a constant NS B-field. We will adopt the notation and conventions of [3], since we want to discuss the S-dual of these theories in the next section. Before we identify the decoupled U(1) states in the string description of these theories, we give a very brief review of the decoupling limit [2] as it applies here.

To define a non-commutative gauge theory as a limit of world volume theory of a 3-brane in string theory, we send $\alpha' \to 0$ while keeping the NS B-field, the open string metric, G^{MN}, and the open string coupling constant, G_o^2, fixed. The open string metric is taken to be the flat Minkowski metric and the NS B-field is non-zero, $B_{23} = -B_{32} = 1/\theta$. In order to keep the open string quantities finite, we have to scale the closed string metric $g_{ij} = (2\pi\alpha'/\theta)^2\delta_{ij}$, $i,j = 2,3$, and the closed string coupling as $g_s = G_o^2(2\pi\alpha'/\theta)$. The decoupled theory of the massless open strings on a 3-brane is then a U(1) gauge theory with non-commutativity in the 2-3 plane. To obtain the theory discussed in the previous section we take the decoupling limit of a single 3-brane, identify $x_{2,3} \to x_{2,3} + 2\pi R$, and put $\theta/(2\pi R^2) = \Theta = 1/s$.

For later reference we note that turning on the NS B-field in the presence of a 3-brane induces D-string charge on it. This in turn is a source for the RR B-field, so a non-zero B_{23}^{NS} induces a non-zero B_{01}^{RR} as can be seen e.g. from the supergravity solutions [17] or from the Chern-Simons term in the 3-brane action. The RR field will not affect the analysis in this section, but it will play a role when we discuss the S-dual theory.

It turns out to be easy to identify the massive non-interacting states discussed in the previous section. Consider an open string state with momentum $p^j = n^j/R$, $j = 2,3$. According to [14], a string moving in one non-commutative direction has a finite length in the orthogonal direction, $\Delta x_i = \theta_{ij}p^j$. Using $\theta_{ij} = (2\pi R^2/s)\epsilon_{ij}$, we see that an open string with s units of momentum in one compact direction is long enough to wind around the other compact direction once. Note that this works only if s in an integer.

It is tempting to identify these states with the non-interacting massive states discussed in the previous section, but there are several problems with that. One can object that there does not seem to be anything special about these open string states, so one should expect them to behave in the same way as other open string states with momenta that are not multiples of s. These states do not in general decouple in the $\alpha' \to 0$ limit, since they give rise to the interacting U(1) degrees of freedom.

Another problem is that the endpoints of open strings with s units of momentum coincide. It seems likely that they can fuse, turning the open string into a closed string. However, closed strings are expected to decouple in the limit of [2]. We will discuss the closed string spectrum first and then argue that open strings with momenta that are multiples of s decouple.

The mass of a closed string state is given by

$$m^2 = \frac{1}{\alpha'^2} g_{ij}(v^i v^j + w^i w^j R^2) + \frac{2}{\alpha'}(N + \tilde{N} - 2)\,, \tag{3.1}$$

where g_{ij} is the closed string metric in the compact directions, w^i is the winding number and

$$v^i = \alpha' g^{ij} \left(\frac{n_j}{R} + 2\pi B_{jk} w^k R \right) . \tag{3.2}$$

In the $\alpha' \to 0$ limit, closed string states with $v^i \neq 0$ or states with excited oscillators become infinitely massive. Only states with $v^i = 0$ and no string excitations survive the decoupling limit. Using $B_{jk} = (1/\theta)\epsilon_{jk} = s/(2\pi R^2)\epsilon_{jk}$ we find that states satisfying $n_j + s\epsilon_{jk}w^k = 0$ remain at finite mass in the decoupling limit. Since both n_j and w^k are integers, this equation has solutions only if s is an integer (or rational number). It is easy to check that these states have the same mass as the open string states discussed above and the same quantum numbers as the non-interacting states we obtained using Morita equivalence in the previous section.

In order to show that the open string states with momenta that are multiples of s decouple we consider the scattering amplitude for four open string states on the 3-brane. In the commutative case the answer is well known [18] and and can be obtained from the type I four-point amplitude of open string states (see e.g. [19]). The main difference between the commutative and non-commutative cases are some additional phases that appear when the positions of vertex operators are switched [9, 14, 20].

Before taking the $\alpha' \to 0$ limit the boundary correlation function in the presence of an NS B-field reads

$$\left\langle x^i(\tau) x^j(\tau') \right\rangle = -2\alpha' G^{ij} + \frac{i}{2}\theta^{ij}\epsilon(\tau - \tau') . \tag{3.3}$$

The l-th vertex operator contains a factor $\exp(ik_l \cdot x(\tau))$, where k_l, $l = 1, \ldots, 4$ denotes the momentum of the l-th open string state. Moving the l-th vertex operator past the m-th generates a phase $\exp(ik_l\theta k_m)$. The complete scattering amplitude for four open string states is a sum of six terms that differ by the ordering or the vertex operators on the boundary of the worldsheet. We write schematically

$$\begin{aligned} A = {} & \{1,2,3,4\} + e^{i\phi_2}\{4,2,3,1\} + e^{i\phi_3}\{2,4,3,1\} + \\ & + e^{i\phi_4}\{1,3,2,4\} + e^{i\phi_5}\{4,3,2,1\} + e^{i\phi_6}\{1,3,4,2\} , \end{aligned} \tag{3.4}$$

where the numbers indicate the ordering of the vertex operators on the boundary of the world sheet and the phases are given by

$$\begin{aligned} &\phi_2 = k_1\theta k_4 , \qquad \phi_3 = k_1\theta k_4 + k_2\theta k_4 , \\ &\phi_4 = k_2\theta k_3 , \qquad \phi_5 = k_2\theta k_3 + k_1\theta k_4 , \qquad \phi_6 = k_2\theta k_3 + k_4\theta k_3 . \end{aligned} \tag{3.5}$$

Recall that the $2,3$ components of the external momenta are quantized in units of $1/R$ and that $\theta_{ij} = (2\pi R^2/s)\epsilon_{ij}$. To show that states with momenta that are multiples of s decouple, we need to show that the all amplitudes involving one or more of these states vanish. We take the $2,3$ components of k_4 to be given by $q_2 s/R$ and

$q_3 s/R$, respectively, and leave the other momenta unspecified. This guarantees that the phases ϕ_2 and ϕ_3 are integer multiples of 2π, i.e. trivial phases. The remaining phases are all given by $ik_2\theta k_3$ up to integer multiples of 2π. The first three terms in eq. (3.4) give the same contribution as the last three up to the over all phase, so we find that for these states the amplitude is related to the commutative amplitude by

$$A_{NC} = \frac{1}{2}\left(1 + e^{i\phi_4}\right) A_C \,. \tag{3.6}$$

However, the commutative amplitude vanishes in the $\alpha' \to 0$ limit, reflecting the fact that there are no interactions in a commutative U(1). This implies that any amplitude with a state that carries momenta that are multiples of s/R vanishes. This is not true in general. If the phases multiplying the various terms in eq. (3.4) are all different, the $\alpha' \to 0$ limit gives a non-vanishing result. Note again, that this cancellation of the phases is possible only if s is an integer.

4. The S-dual picture

Having identified the non-interacting massive states in the non-commutative U(1) theory as strings with specific winding and momentum quantum numbers, we can now proceed to search for these states in the S-dual NCOS description [3, 4].

Before delving into details, let us outline the qualitative picture of both the non-commutative U(1) and its S-dual. Turning on the NS B-field induces D-string charge on the 3-brane. If $\Theta = 1/s$ the induced charge corresponds to s D-strings, and these branes are a source for the RR B-field. Following [3], we can S-dualize the non-commutative U(1) theory. This turns the induced D-strings into fundamental strings corresponding to electric flux on the 3-brane, and exchanges the role of the NS and RR B-fields. In the non-commutative U(1) both B_{23}^{NS} and B_{01}^{RR} were non-zero. In the S-dual theory we have $B_{23}^{\mathrm{RR}} = -1/\theta$ and B_{01}^{NS}, or, equivalently, the electric flux on the 3-brane can be computed as follows. The electric flux is given by the solution of

$$\frac{\sqrt{-\hat{g}}}{\hat{g}_{str}} \frac{\hat{g}^{00}\hat{g}^{11}F_{01}}{\sqrt{1 + (2\pi\alpha')^2\hat{g}^{00}\hat{g}^{11}F_{01}^2}} = B_{23} = \frac{s}{2\pi R^2}\,, \tag{4.1}$$

where the closed string metric is given by $\hat{g}_{11} = -\hat{g}_{00} = \theta/(2\pi\alpha' G_o^2)$ and $\hat{g}_{22} = \hat{g}_{33} = 2\pi\alpha'/(G_o^2\theta)$. The hat over symbols denotes the S-dual quantities.[1] This gives using $\hat{g}_{str} = R^2/(\alpha' G_o^2 s)$

$$F_{01} = \frac{F_{01}^c}{\sqrt{1 + (\alpha' s/R^2 G_o^2)^2}}\,, \qquad F_{01}^c = \frac{2\pi R^2}{s}\frac{1}{G_o^2(2\pi\alpha')^2}\,. \tag{4.2}$$

[1]The symbols without hats in the first part of the paper correspond to the primed quantities in [3], and the symbols with hats are the unprimed variables in [3].

In the limit $\alpha' \to 0$ the electric field attains its critical value F^c_{01}, but the resulting theory on the 3-brane is not a field theory, which would be non-unitary [21]. Instead it turns out to be a open string theory with non-commutativity in the 0-1 plane (NCOS). This can be seen most easily by trading the electric flux on the 3-brane for a non-zero $B^{\rm NS}_{01}$ in the bulk.

We want to identify the non-interacting states discussed in the previous two sections in the NCOS and verify that they do not interact. The states are easy to identify. Since they correspond to fundamental strings with winding and momentum in the compact directions in the string description of the non-commutative U(1), they should turn into D-strings with winding and momentum in the S-dual picture.

We have to consider two different wound D-strings, D-strings with w^3 units of winding in the 3-direction and sw^3 units of momentum in 2 and states with winding number w^2 in 2 and $-sw^2$ units of momentum in 3. For the purpose of counting the unbroken supersymmetries we can introduce new coordinates

$$\begin{pmatrix} x'_2 \\ x'_3 \end{pmatrix} = \frac{1}{(w^2)^2 + (w^3)^2} \begin{pmatrix} w^3 & -w^2 \\ w^2 & w^3 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}. \tag{4.3}$$

An arbitrary superposition of the D-string states is represented as a state with one unit of winding in the $3'$ direction and $-s$ units of momentum in the $2'$ direction. In the subsequent analysis we will drop the primes and only consider states with $(n_2, w^3) = (-s, 1)$.

It is very difficult to show that D-string states decouple from the dynamics of the open strings of the NCOS, but we can provide some circumstantial evidence that our identification of the decoupled heavy states is correct. We can compute the masses of these D-string states and we can show that they are 1/4 BPS states in the presence of the 3-brane with electric flux.

To address the issue of supersymmetry we can ignore the background RR B-field. It shifts the masses of some states, but does not modify the supercharges preserved by the individual branes. One way to see this is to note that the RR B-field can be gauged away locally. Unlike in the case of an NS B-field, this transformation does not induce any fieldstrenghts on the brane, so the unbroken supersymmetries will not depend on the RR B-flux through the torus.

So, for the purpose of counting the unbroken supersymmetries we set $B^{\rm RR}_{\mu\nu} = 0$. We emphasize that this is a computational tool and that the physics of the D-string does depend on the values of the RR B-field. To construct the unbroken supersymmetries we need to compute the velocity of a D-string with momentum $p = -s/R$ in the 2-direction. The canonical momentum can be computed from the lagrangian for a D-string written in terms of the open string quantities

$$L = \frac{2\pi R}{2\pi\alpha' \hat{G}_o^2} \sqrt{-\hat{G}_{00}\hat{G}_{33}} \sqrt{1 - v_2^2}\,, \tag{4.4}$$

where v_2 is the velocity in the 2 direction and $\hat{G}_{AB} = 2\pi\alpha'/(G_o^2\theta)\eta_{AB}$.[2] Equating the canonical momentum with $p = -s/R$, solving for v_2 and using $\theta = 2\pi R^2/s$ we find $v_2 = 1/\sqrt{2}$.

One way to count the supersymmetries preserved by s fundamental strings in directions 01 bound to a D3-branes in 0123, and a D-string with one unit of winding in 3 and $-s$ units of momentum in 2 is to use T-duality of type-IIB string theory to convert all objects into D-branes. To this end we compactify all spatial directions of the 3-brane and T-dualize the 1 and 3 directions. This turns the original D-string into another D-string in 01 with $-s$ units of momentum in 2. The original 3-brane turns into a D-string in 02 with s units of momentum in 1. We can count the supersymmetries of this configuration using standard techniques [19]. The only modification arises from the relative motion of the two branes, which can be taken into account by boosting the supercharges appropriately. Using the Born-Infeld action it is a simple matter to check that the D-string in 02 with s units of momentum in 1 moves with $v_1 = -1/\sqrt{2}$ and we already computed $v_2 = 1/\sqrt{2}$ above. The supercharges left unbroken by the two D-strings separately are

$$Q_L + \Lambda_{1/2}(v_1)\Gamma_2^{\perp}\Lambda_{1/2}^{-1}(v_1)Q_R\,, \qquad Q_L + \Lambda_{1/2}(v_2)\Gamma_1^{\perp}\Lambda_{1/2}^{-1}(v_2)Q_R\,, \tag{4.5}$$

where $\Lambda_{1/2}(v_i)$ is the matrix for a boost in the i direction with parameter v_i acting on spinor indices, and $\Gamma^{\perp} = \prod_{\mu\in\perp}\Gamma\Gamma^{\mu}$ is obtained by taking the product over all transverse directions (see [19] for details).

By explicit calculation we find that the configuration of two D-strings we consider here preserves a quarter of the supersymmetries. Note that this is true only if the D-strings carry s units of momentum with the appropriate signs. In all other cases the two conditions in eq. (4.5) are incompatible and the superposition of these D-strings preserves no supersymmetry. This does not mean that they cannot relax into a supersymmetric ground state, but it does imply that the system can lower its energy. Such an instability is usually signaled by a tachyon in the open string spectrum. The main observation here is that for states with s units of momentum there is no tachyon and the superposition of the two D-strings is a 1/4 BPS state. Undoing the two T-dualities we conclude that the D-string states we consider here form a marginally bound state with the 3-branes with s units of electric flux, provided they have compact momenta that are multiples of s. All other D-string states have an instability against dissolving in the 3-brane.

Having counted the supersymmetries preserved by this configuration we are now in position to compute the mass of these D-string states. The mass does depend on the RR flux through the torus. Consequently we restore the RR B-field to its original value $B_{23}^{\mathrm{RR}} = -1/\theta$ and add the Chern-Simons term

$$L_{\mathrm{cs}} = -2\pi R B_{23}^{\mathrm{RR}} v_2 \tag{4.6}$$

[2]Reference [3] uses the symbol $\tilde{G}_{AB}$ for this metric.

to the lagrangian of the D-string equation (4.4). Computing the canonical momentum from the complete action and solving for the velocity of the D-string now yields $v_2 = 0$. Using this result we readily verify from eq. (4.4) that the mass of a D-string with one unit of winding and s units of momentum is $M = s/R$ as expected. We emphasize that these states have the correct mass only if the RR B-field is included. Setting these fields to zero is valid in the non-compact case, but if any of the NCOS directions are compactified, it is essential to retain both the NS and the RR B-fields.

While it is straightforward to show that the D-string states discussed above are 1/4 BPS states as expected from the dualities, it is much more involved to show that they do not interact. To show that these states decouple from the NCOS degrees of freedom, we would have to show that the amplitude for two NCOS strings to scatter into these D-string states vanishes. This is a non-perturbative calculation and as such it is beyond out present capabilities. We would also need to verify that the scattering amplitude for two of these D-string states in the presence of the 3-brane with s units of magnetic flux vanishes. Both of these computations are very difficult and we will not attempt them here.

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